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Estimating number of simultaneously yielding stories in a shear building subjected to full-sine pulse velocity base excitation

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ABSTRACT

Past research has shown that seismic force demands in the columns of lateral load resisting systems depend on the number of simultaneously yielding stories (N_{SYS}) above the column considered. Current design procedures for steel frames (e.g., those in AISC-341) typically specify that all stories are considered to be yielding. Time history analyses show that this assumption may be overly conservative for tall buildings. No systematic procedure exists for estimating this N_{SYS}. Concepts of wave propagation theory are used here in seeking such a procedure. As a first step towards that goal, research was conducted to investigate the N_{SYS} due to an incident wave in a shear-type building subjected to full-sine velocity base excitation, to understand the relationships between input excitation, inter-story drifts, story forces, and hence to formulate a procedure to estimate the N_{SYS} along the building height. Story yielding capacity of a base excitation was found to depend on the magnitude of its velocity record. Accordingly, a parameter v_y was defined that determines the minimum magnitude of the velocity wave required to yield a story. Mathematical expressions were derived to predict the beginning and end of story yielding due to the incident wave. With this, N_{SYS} values at any instant as the incident wave propagates up the building could be determined. This estimation procedure was found to provide good estimates of the actual yielding for structures considered here that have v_y values decreasing with height, with the estimated values being on the conservative side.

1. Introduction

In design procedures based on capacity design principles, such as the AISC 341-10 Seismic Provisions for Structural Steel Buildings [1], the axial force demand in columns of a seismic force resisting system (SFRS) is often specified to be obtained by considering that all the ductile members along the height of the SFRS have yielded simultaneously. However, time history analyses show that although occurrence of simultaneous yielding over the entire height of a building may occur in low-rise structures, it is not necessarily the case for mid to high-rise structures. Studies on seismic demands in columns have been done and techniques for estimating column demands have been proposed in the past (e.g., by Redwood and Channagiri [2], Tremblay and Robert [3], Lacerte and Tremblay [4], Richards [5], to name a few). Many of these studies report that maximum column axial forces occur when the plastic yielding mechanism is developed over multiple consecutive stories above the level under consideration. For example, for braced frames, Redwood and Channagiri [2] proposed that the axial force demand in the column at a particular level be obtained by adding the vertical component of the ultimate brace forces immediately above that story, to the SRSS of vertical forces from all the other braces above that level, while Lacerte and Tremblay [4] proposed a method that considered the forces in critical braces as 1.0 and 1.2 times the expected brace capacity in tension and compression, respectively, and brace forces in all the other floors as those obtained when brace buckling initiates in these stories. Richards [5] investigated the demands on seismic columns of various types of braced frames considering four different system strengths and three different structural heights and reported that performing capacity design assuming simultaneous yielding over the entire height of tall buildings was in some cases overly conservative. These studies have typically been empirical, relying on nonlinear time history analysis of a number of typical frames. While these have provided valuable (but limited) insights on seismic demand in columns, there is much to gain by developing a

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systematic and generalized way to estimate the expected number of simultaneously yielding stories (N_{SYS}) in building frames, because this is an essential step towards being able to determine column forces from a modified capacity design perspective. In absence of such a procedure, the capacity-design approach as implemented in current design procedures could severely overestimate the actual axial demands on columns, resulting in overdesigned and economically inefficient columns.

Towards the above goal, the authors undertook to investigate whether wave propagation theory could be used to develop a systematic procedure for estimating the number of simultaneously yielding stories, and, eventually, axial force demand in columns. Three essential steps are envisioned for this purpose: First, a procedure must be developed for estimating the number of simultaneously yielding stories in a simple shear building subjected to velocity-pulse base excitation; Second, this procedure must be adapted as necessary for shear buildings subjected to actual earthquake excitations, in the perspective that earthquakes can be represented as a series of pulses, and; Third, a procedure must be formulated to estimate the axial force demand in columns considering the force transferred from the simultaneously yielding stories and the other non-yielded stories above the column under consideration.

This paper focuses only on the first of these steps, by investigating the number of simultaneous yielding stories due to an incident wave in a tall shear type building subjected to full-sine velocity base excitation, to understand the relationships between input excitation, inter-story drifts, story forces, and hence to formulate a procedure to estimate the number of simultaneously yielding stories along the building height. Although some researchers have previously used wave propagation analysis or aspects of wave propagation theory to find or understand the response of structures due to pulses and earthquakes (e.g., Clough and Penzien [6], Humar [7], Safak [8], Hall et al. [9], Krishnan and Muto [10]) and for system identification and health monitoring (e.g. Sneider and Safak [11], Todorovska and Trifunac [12], Todorovska and Trifunac [13], Todorovska and Rahmani [14], Ebrahimian and Todorovska [15]), the concepts and approach presented here to obtain the number of simultaneously yielding stories and axial force demand in columns is novel.

Note that tall structures like those considered in the study presented here are also subjected to flexural deformations. However, in this initial work, buildings deforming only in shear are considered. Using such a simple model allows to investigate the validity and potential of the concepts before considering applications to more complicated structures.

2. Concepts of wave propagation to find number of simultaneously yielding stories

2.1. Magnitude of velocity wave required to cause story yielding

A ground displacement is taken here as an excitation at the base of a building, generating a traveling wave along its height. Based on classical wave propagation theory, for a shear building with uniform story height and mass, with no damping, and within elastic range, displacement at any point x and instant t can be expressed as a combination of forward and backward propagating displacement waves along the height of the building. Here, the forward and backward traveling displacement waves have been denoted as u_f and u_b, respectively, as shown in Eq. (1).

$$u(x,t) = u_f(x - ct) + u_b(x + ct) \tag{1}$$

where, c is the velocity of a propagating wave. Similarly, shear strain s at any point can also be expressed as a summation of forward and backward moving waves as shown in Eq. (2) and can be expressed in terms of displacement waves as shown in Eq. (3).

$$s = s_f + s_b \tag{2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u_f}{\partial x} + \frac{\partial u_b}{\partial x} \tag{3}$$

The corresponding velocity v, and forward and backward velocity waves v_f and v_b are expressed as in Eq. (4), and related to displacement and shear strain as per Eqs. (5) and (6), respectively.

$$v = v_f + v_b \tag{4}$$

or,

$$\frac{\partial u}{\partial t} = \frac{\partial u_f}{\partial t} + \frac{\partial u_b}{\partial t} \tag{5}$$

or,

$$\frac{\partial u}{\partial t} = -c\frac{\partial u_f}{\partial x} + c\frac{\partial u_b}{\partial x} = c(-s_f + s_b) \tag{6}$$

For buildings having shear-type lateral-load resisting systems (a.k.a. shear buildings or shear frames), the corresponding elastic shear force induced at a particular story in a building is determined by Eq. (7), where V is the story shear force, K is the shear stiffness, and $\frac{\partial u}{\partial x}$ is the shear strain at that story.

$$V = K \times \frac{\partial u}{\partial x} \tag{7}$$

Story yielding occurs when the story shear force reaches its yield capacity V_p. If the corresponding shear strain is represented as $\left(\frac{\partial u}{\partial x}\right)_y$, then the force deformation relationship, at the onset of story yielding, can be expressed as:

$$V_p = K \times \left(\frac{\partial u}{\partial x}\right)_y \tag{8}$$

or, rearranging Eq. (8),

$$\left(\frac{\partial u}{\partial x}\right)_y = \frac{V_p}{K} \tag{9}$$

Also, if k is the story stiffness and h is the story height, shear stiffness K can be expressed as:

$$K = k \times h \tag{10}$$

Then, Eq. (9) can be written as:

$$\left(\frac{\partial u}{\partial x}\right)_y = \frac{V_p}{kh} \tag{11}$$

Thus, $\left(\frac{\partial u}{\partial x}\right)_y$ is the shear strain required to yield a story with story stiffness of k and shear yield capacity of V_p.

Shear strain can also be expressed as a combination of forward and backward moving velocity waves expressed in Eq. (12).

$$\frac{\partial u}{\partial x} = \frac{1}{c}\left(-\frac{\partial u_f}{\partial t} + \frac{\partial u_b}{\partial t}\right) = \frac{1}{c}(-v_f + v_b) \tag{12}$$

Substituting Eq. (12) in Eq. (11) and rearranging the terms, one obtains:

$$(-v_f + v_b)_y = \frac{c}{h}\frac{V_p}{k} \tag{13}$$

The left hand side can be denoted as v_y since it gives a measure of the minimum magnitude of the linear combination of the forward and backward moving velocity waves required to cause yielding in the story and it depends on the structural property of the building under consideration. Denoting the time taken by the wave to travel through the story with height h as t_h, then v_y can also be expressed as:

$$v_y = (-v_f + v_b)_y = \frac{V_p}{kt_h} \tag{14}$$

Since,

$$c = \sqrt{\frac{kh}{m}} \tag{15}$$

where, m is the story mass. Time taken by the wave to travel through story height h can be calculated using Eq. (16).

$$t_h = \frac{h}{c} = \sqrt{\frac{mh}{k}} \tag{16}$$

The advantage of expressing shear strain in terms of velocity in Eq. (12) and obtaining a measure of story yielding in terms of velocity in Eq. (14), is that it enables to readily use the ground motion velocity input to determine its potential to cause yielding in the stories, as will be shown below.

Since shear force can also be expressed as a product of story stiffness k and inter-story drift Δu,

$$V = k \times \Delta u \tag{17}$$

Then, from Eqs. (7), (10), (12), and (17)

$$\therefore -v_f + v_b = \frac{V}{kt_h} = \frac{\Delta u}{t_h} \tag{18}$$

The summation of the backward moving velocity wave and the negative value of the forward moving velocity wave (i.e., $-v_f + v_b$) can also be expressed as $\Delta u/t_h$, and will be referred to as the $\Delta u/t_h$ wave.

2.2. Estimating number of simultaneously yielding stories

As a $\Delta u/t_h$ wave enters a story, yielding initiates when the value of the wave at that story exceeds its v_y value. As the wave propagates further, the story continues to yield. For simplicity, it is assumed that yielding at a story occurs as long as the value of the wave exceeds the v_y value of the story and stops once the value of the wave decreases below the v_y value. For example, if story yielding due to the incident wave of the full-sine velocity base excitation shown in Fig. 1 is considered and assuming that the shape and magnitude of the wave does not change as it propagates, a particular story j will start to yield when point A along the velocity wave, A being the point at which the value of the wave is equal to the v_y value of the j^{th} story, reaches that story. As the wave propagates, it continues to yield until point B along the velocity wave, after which the value of the wave is less than the v_y value of the story, reaches it. Therefore, story j yields for a duration of $t_{yd,j}$ as indicated in the figure. The story yielding actually does not end exactly at point B, but somewhere close to it; thus, yield duration $t_{yd,j}$ is an approximate value. Note that the assumption mentioned above relates to the end of story yielding and will only affect the prediction of duration of story yielding.

Yield capacity and story stiffness typically vary for different stories in the building; this will result in different v_y values at various stories of the building. As the velocity waves propagates along the building height, yielding will occur in all the stories that are encompassed within the span of the $\Delta u/t_h$ wave and for which the v_y value is smaller than the value of the $\Delta u/t_h$ wave (i.e., value of "$-v_f + v_b$") at that corresponding stories. Thus, at any instant, as the wave is traveling along the building height, comparing the value of $(-v_f + v_b)$ and the v_y value at all the stories falling within

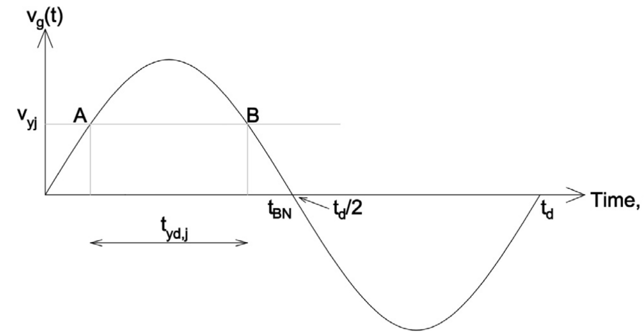

Fig. 1. Points along the velocity wave corresponding to the beginning and end of yielding at j^{th} story.

the wavelength should give an estimate of the number of stories yielding simultaneously at that time. For example, consider an instant when the input velocity motion at the base (without considering change in its shape due to various effects) has reached the 16th floor of a twenty story building as the incident velocity wave shown in Fig. 2. At this instant, stories 6 to 16 fall within the wavelength of the velocity pulse. However, stories 7 to 9 and 12 to 15 have v_y values lower than the magnitude of the velocity wave at the respective floors. Thus, at that instant, only stories 7, 8, 9, 12, 13, 14, and 15 will yield simultaneously, as indicated by the red[2] and blue solid circles in the figure. The yielding represented by the red and blue solid circles are due to the first and second part of the full-sine velocity wave respectively. Note that the yielding caused by these two parts are of opposite sign. So, the axial force induced by these two cases will be of opposite signs. Here, yielding due to first part only is considered as it provides results on the conservative side.

Note that as the wave propagates up the building, the shape of the wave changes due to various factors, such as discontinuity in system properties, inherent damping, and inelasticity. Here, the number of stories yielding simultaneously is estimated by considering the original shape of the velocity base excitation, assuming that the waves do not change in shape and magnitude as they propagate along the building height. This assumption, together with using velocity base excitation for the estimation procedure was found to be reasonable for a structure having v_y values decreasing with height, as expected in regular building structures.

2.3. Potential of ground motion to cause yielding in a story and instances of simultaneous story yielding

The potential of a ground motion to cause story yielding depends on the magnitude of the ground motion velocity input and the v_y values at respective stories. If the amplitude of the velocity input excitation is greater than the v_y value at any story, then the structure will yield when the incident wave first travels from the base to the top of the building. However, due to constructive overlaps of $-v_f$ and v_b waves, the magnitude of the resulting $\Delta u/t_h$ wave may be large enough to cause yielding of the stories falling within the overlapping region. Constructive phase overlapping for the full-sine velocity base-excitation wave considered here occurs only at two locations, namely, at the base and at one-fourth of the wavelength from the free end. Thus, simultaneous yielding across multiple stories can occur in three instances: (i) due to incident wave traveling up the building (Fig. 3a), (ii) due to

[2] For interpretation of color in Figs. 2 and 10, the reader is referred to the web version of this article.

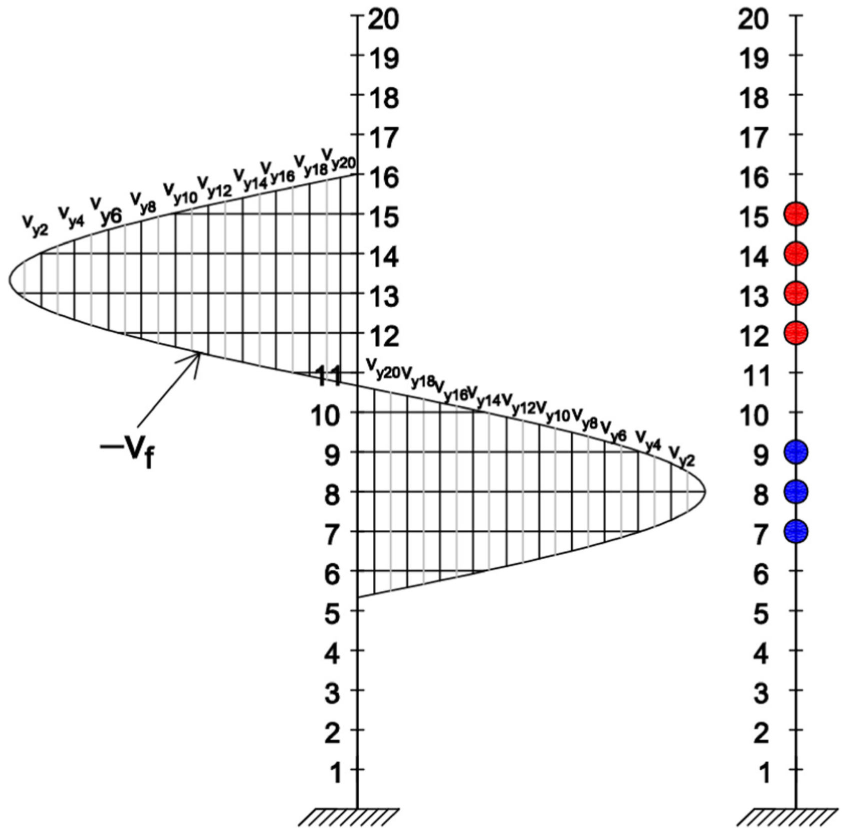

Fig. 2. Simultaneously yielding stories due to incident velocity wave.

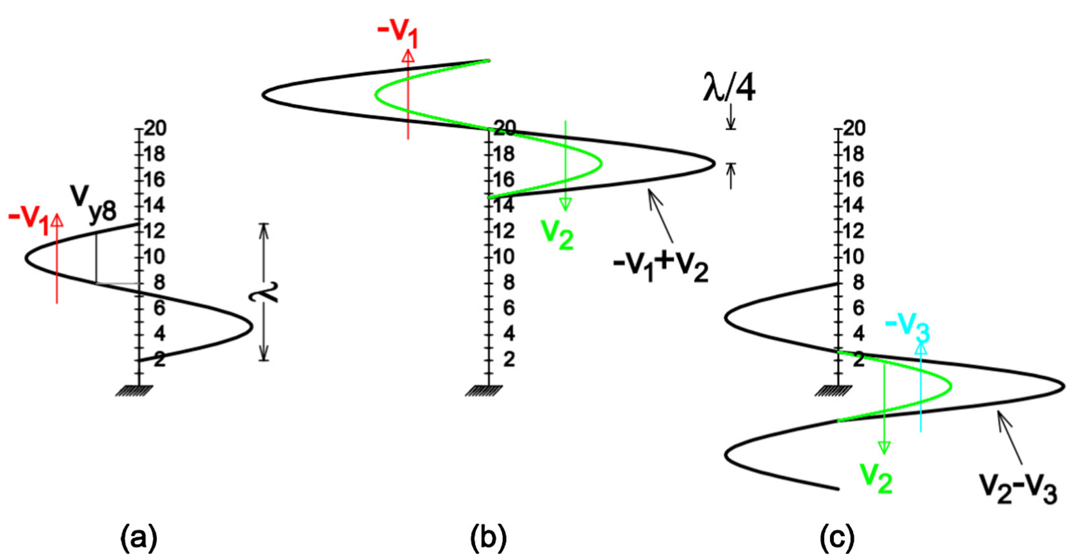

Fig. 3. Conditions that need to be considered while estimating axial force demands in columns.

constructive overlap of the $-v_f$ and v_b velocity waves at $\lambda/4$ from the top of the building (Fig. 3b), and (iii) due to constructive overlap of the $-v_f$ and v_b velocity waves at the base of the building (Fig. 3c). Here, λ is the wave length of the full-sine velocity pulse used in the study. Note that the overlapping of the velocity waves described above is for waves propagating along elastic medium. If the structure undergoes inelastic deformation, the shape and magnitude of the waves will change. Thus, the overlapping of the wave will not be exactly as illustrated in Fig. 3, but similar to that.

3. Analysis parameters

3.1. Input excitation

To study the effect of short and long pulse durations, ratios of pulse duration to fundamental time period of the building, t_d/T_n, of 0.2 and 0.8 have been considered. The fundamental time period of the building, T_n, is 3.29 s. Hence, the time periods of the short and long duration pulses used are 0.658 s and 2.632 s respectively.

Amplitudes of both pulses were chosen to be same. The full-sine base excitation velocity pulse, along with its corresponding acceleration and displacement base motions, are shown in Fig. 4.

Note that the structural response of concern here, namely the number of simultaneously yielding stories, depends on the ratio of t_d by T_n. As t_d/T_n increases, the number of simultaneously yielding stories increases. More correctly, the number of simultaneously yielding stories increases with increase in height of the building over which the pulse spans which is measured by the t_d/t_H ratio. Here t_d is the pulse duration and t_H is the time taken by the wave to travel through the height of the building. The change in fundamental time period or the change in pulse duration can change the number of simultaneously yielding stories in the building.

3.2. Structural system

A simple forty story shear building has been chosen for the study. The model essentially consists of columns, with degrees of freedom at the nodes corresponding to story levels restrained such that they can only undergo lateral translation (i.e., corresponding to frame shear deformation only). The story masses are lumped at story levels. The behavior of this model is analogous to that of a stick frame-model of the structure with infinitely rigid beams and flexible columns. The columns are assigned the required flexural stiffness to represent the assumed story shear stiffness. Fig. 5 shows the model used for study along with the mode of deformation and degrees of freedom. The structure is considered to be made up of steel with yield strength of 50 ksi [345 MPa]. and modulus of elasticity of 29,000 ksi [200 GPa]. The structure has a story height of 10 ft [3.05 m] and story weight of 1.02 kips [4.54 KN]. Two types of variation in story stiffness over the height have been considered as explained below. In both the cases, the buildings have fundamental time period, T_n, of 3.29 s.

To get a perspective on the effect of variable story stiffness and elastic-plastic behavior on the structural performance, four different structural systems have been studied. The structural systems considered are as follows:

- Structure-I is an elastic structure with uniform story stiffness of 6.41 kips/in [1123 KN/m] throughout the height of the building.
- Structure-II is identical to Structure-I, except that it is modeled to behave inelastically. Force reduction factor R of 1, 2, 4, and 6 have been used. Three different variations in how the shear yield capacity V_p is distributed over the building height were considered as described in the next section.
- Structure-III is an elastic structure with varying story stiffness over the height of the building. The story stiffness varies from 7.9 kip/in [1383 KN/m] at the bottom story to 0.39 kip/in [68 KN/m] at the top story. Fundamental period of the Structure-III was chosen to be the same as for Structure-I. The story stiffness at different floors have been determined based on the requirement that the fundamental time period of the building remains the same for all cases considered (namely, 3.29 s) and the assumption that the application of static forces as given in codes will induce a floor displacement that increases linearly with height. This will cause equal inter-story drift at all the stories as the story height is uniform throughout the building. This procedure has been explained in Chopra [16]. For detailed calculation of the story stiffness refer to Shrestha and Bruneau [17].
- Structure-IV is identical to Structure-III, except that it is allowed to undergo inelastic deformations. Three different variations of shear yield capacity, V_p, over the height of the building were considered as done for Structure-II.

Here both undamped and damped cases have been considered. For damped case, Rayleigh damping with viscous damping of 2% defined at the first and 20th modes has been used for Structures-I and -II and at the first and 24th modes for Structures-III and -IV. Table 1 presents all the combination of structural system considered for the study.

Note that the number of simultaneously yielding stories depends on the v_y values at the stories and the height of the building over which the velocity wave spans (measured by t_d/t_H ratio). The lower the v_y values and higher the t_d/t_H ratio, the larger will be the N_{SYS} value. Hence, N_{SYS} is inversely proportional to t_H and v_y values at the stories. Since t_H is directly proportional to the time taken by the wave to travel through story height h, N_{SYS} is inversely proportional to t_h and v_y values at the stories as shown in Eqs. (19)–(21).

$$N_{SYS} \propto \frac{1}{t_h} \tag{19}$$

$$N_{SYS} \propto \frac{1}{v_y} \tag{20}$$

Combining Eqs. (19) and (20),

$$N_{SYS} \propto \frac{1}{t_h v_y} \tag{21}$$

Since,

$$t_h = \frac{h}{c} = h\sqrt{\frac{m}{kh}} = \sqrt{\frac{mh}{k}} \tag{22}$$

$$v_y = \frac{V_p}{kt_h} = \frac{V_p}{k} \times \sqrt{\frac{k}{mh}} = \frac{V_p}{\sqrt{mhk}} \tag{23}$$

where, m is the story mass, h is the story height, k is the story stiffness, and V_p is the shear yield capacity of the story.

From Eqs. (21)-(23),

$$N_{SYS} \propto \frac{1}{\frac{V_p}{\sqrt{mhk}} \times \sqrt{\frac{mh}{k}}} \tag{24}$$

or,

$$N_{SYS} \propto \frac{k}{V_p} \tag{25}$$

The mathematical expression in Eq. (25) shows that the N_{SYS} values are actually independent of the mass of the building. In other words, as shown by Eqs. (22) and (23), as mass increases, the time taken by the wave to travel through the building height increases; and hence, the height of the building over which the velocity wave spans decreases. However, an increase in mass also decreases the v_y values. Hence the decrease in N_{SYS} values that would result from the decrease in the height of the building over which the velocity wave spans as a result of increase in mass is exactly equal to the increase in N_{SYS} values that would occur due to decrease in v_y values. On the other hand, Eq. (25) also indicates that as stiffness increases N_{SYS} values increase.

3.3. Distribution of V_p and v_y values

Three different variations of the shear yield capacity V_p over the building height were considered: (a) V_p values based on a code defined lateral force distribution; (b) V_p values varying linearly with height, and; (c) V_p values determined from the maximum elastic force demands induced by the incident wave propagating along the building (Shrestha and Bruneau [17]). The last two variation of V_p, of academic interest, have been considered to provide

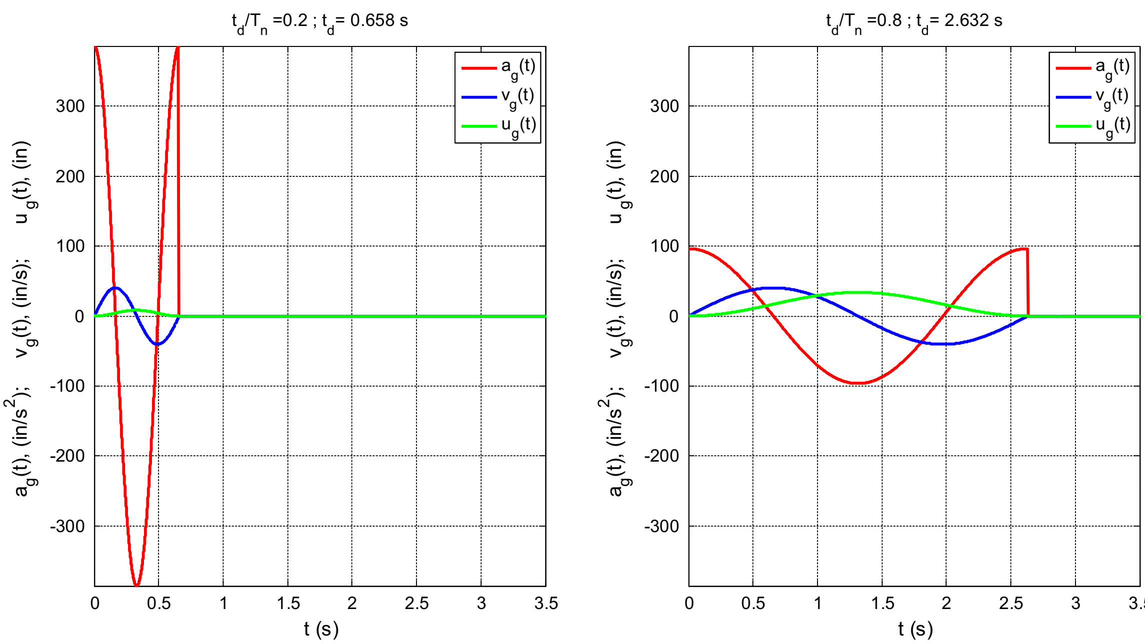

Fig. 4. Input base excitation.

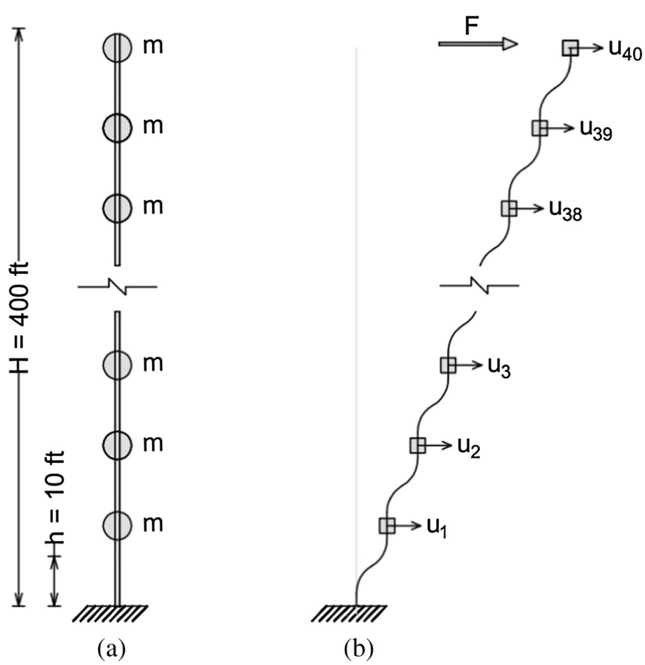

Fig. 5. (a) Model used for study and (b) mode of deformation and degrees of freedom.

further verification of the proposed procedure to predict story yielding and to see how variations in the distribution of V_p over the building height (for other distributions than that based on the code procedure) will change the story yield pattern. Here, only results related to the first type of V_p distribution are provided since it represents the force distribution in real buildings built according to codes. For the case of $R = 1$, V_p at the base has been chosen to be equal to the elastic force demand in that story. For variation of V_p values along the building height based on code, the ratio of shear yield capacity at each stories to the shear yield capacity at the base story, $V_{p,j}/V_{p,base}$, was obtained by considering the code based lateral force distribution of Eq. (26):

$$F_j = \frac{w_j h_j}{\sum_{i=1}^{N} w_i h_i} \times V_b \tag{26}$$

where F_j is the lateral force at the j^{th} story, V_b is the base shear, w_j is the weight of the j^{th} story, h_j is the height of the j^{th} story from the base of the building, and N is the number of stories in the building. Note that for simplicity, the value of the exponent k in the equation for lateral force distribution according to code has been taken here to be 1, which corresponds to a linear distribution of the lateral forces. The shear force at the j^{th} story can then be obtained using Eq. (27).

$$V_j = \sum_{k=j}^{N} F_k \tag{27}$$

Substituting Eq. (26) in Eq. (27), and rearranging, the ratio of shear force at the j^{th} story to that at the base can be expressed as:

$$\frac{V_j}{V_b} = \sum_{k=j}^{N} \left(\frac{w_k h_k}{\sum_{i=1}^{N} w_i h_i} \right) \tag{28}$$

If distribution of the yield shear force is chosen to be the same as that for the elastic shear force given by the code procedure in Eq. (28), substituting V_j by shear yield capacity at the j^{th} floor $V_{p,j}$ and V_b by shear yield capacity at the base story $V_{p,base}$, Eq. (28) can be rewritten to obtain ratio of shear yield capacity at j^{th} story to that at the base as:

$$\frac{V_{p,j}}{V_{p,base}} = \sum_{k=j}^{N} \left(\frac{w_k h_k}{\sum_{i=1}^{N} w_i h_i} \right) \tag{29}$$

Knowing the shear yield capacity at the base and the distribution of the $V_{p,j}/V_{p,base}$ ratio over the building height, shear yield capacity at each floor can be calculated. The distribution of the $V_{p,j}/V_{p,base}$ values at different stories for the cases considered are shown in Fig. 6.

The v_y value at a story depends on the story stiffness k, time taken by the wave to travel through that story t_h, and the corresponding story yield capacity V_p as expressed in Eq. (14). The v_y values obtained for all the distributions of story stiffness considered, viscous damping, and pulse duration of the input ground

Table 1
Structural system considered.

Case	t_d/T_n		Damping		Structure type				Story stiffness		Response		V_p variation over height		
	0.2	0.8	0%	2%	I	II	II	IV	Constant over height	Varying over height	Elastic	Inelastic	Code based	Linear	Wave based
1	×		×		×				×		×				
2	×		×			×				×		×	×		
3	×		×			×				×		×		×	
4	×		×			×				×		×			×
5	×		×				×		×		×				
6	×		×					×		×		×	×		
7	×		×					×		×		×		×	
8	×		×					×		×		×			×
9	×			×	×				×		×				
10	×			×		×				×		×	×		
11	×			×		×				×		×		×	
12	×			×		×				×		×			×
13	×			×			×		×		×				
14	×			×				×		×		×	×		
15	×			×				×		×		×		×	
16	×			×				×		×		×			×
17		×	×		×				×		×				
18		×	×			×				×		×	×		
19		×	×			×				×		×		×	
20		×	×			×				×		×			×
21		×	×				×		×		×				
22		×	×					×		×		×	×		
23		×	×					×		×		×		×	
24		×	×					×		×		×			×
25		×		×	×				×		×				
26		×		×		×				×		×	×		
27		×		×		×				×		×		×	
28		×		×		×				×		×			×
29		×		×			×		×		×				
30		×		×				×		×		×	×		
31		×		×				×		×		×		×	
32		×		×				×		×		×			×

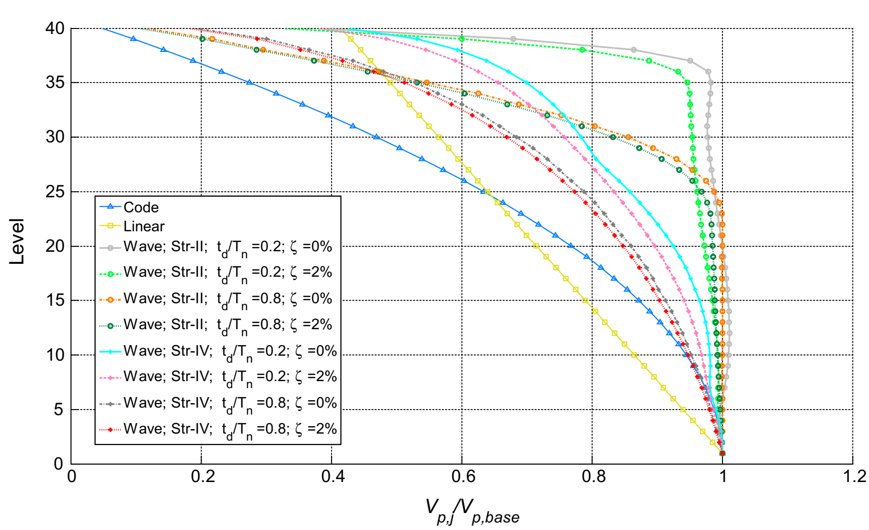

Fig. 6. Distribution of $V_{p,j}/V_{p,base}$ ratio over the building height of Structure-II and -IV for the cases of different variation of V_p values over the building height (i.e., V_p values based on code, V_p values varying linearly, and V_p values based on wave).

motion are shown in Fig. 7. It is known that the presence of damping causes a decrease in the magnitude of the velocity wave and hence the shear force as it propagates. Since V_p at the stories here depend on the elastic demand at the base, the V_p values at the stories also decrease accordingly. This causes the v_y values at the stories to decrease as well when damping is introduced. If magnitude of the pulse is held constant, when the pulse duration is increased, the inter-story drift decreases, which will also decrease the shear forces at the stories. Hence, as explained before since the V_p values depend on the elastic demand at the base, when pulse duration is increased the V_p values decrease and so do the v_y values.

As mentioned before, Table 1 presents all the combination of structural system considered for the study. Note that though analysis of all the structural cases presented in the table was conducted and their response was studied, in this paper only inelastic structures (i.e. Structure-II and -IV) with two percent viscous damping

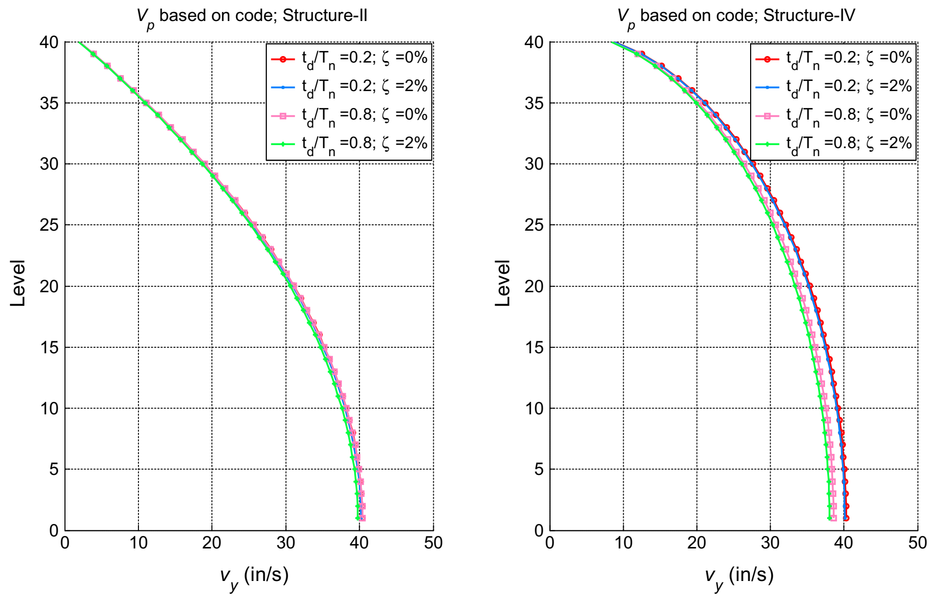

Fig. 7. Distribution of v_y values at different stories for Structures-II and -IV with V_p variation based on code subjected to short duration (t_d/T_n = 0.2) and long (t_d/T_n = 0.8) duration pulses.

and distribution of V_p values based on code defined lateral force distribution are presented (refer to Shrestha and Bruneau [17] for detailed analysis results of all the structural cases presented in Table 1).

3.4. OpenSees model

Dynamic analysis was conducted using the Open System for Earthquake Simulation (OpenSees) analysis software. For elastic analyses (i.e., Structure-I and Structure-III), an elastic beam-column element was used to model the columns at each story, while for inelastic analyses (i.e., Structure-II and Structure-IV), a nonlinear beam-column element was used. For the inelastic cases, linear elastic material property was defined for axial force, while uniaxial bilinear material property was defined for flexure. Kinematic hardening with a strain hardening ratio of 1% was considered. The section was then defined by combining the flexural and axial properties using the "section aggregator" command in OpenSees, considering no interaction between axial and flexure properties. The structure was fixed at its base. Axial translational and rotational degrees of freedom at each floor were restrained, allowing only lateral translation, to model the structure as a shear type building.

For the OpenSees analysis, transient analysis was conducted using transformation method for constraint handler, Plain degree-of-freedom numbering object, BandGeneral SOE linear system of equation object, Norm Displacement Increment test for convergence test, Linear algorithm to solve non-linear equations, and Newmark Method for integrator object. To record the outputs, node recorder was used for nodal displacements and element recorder was used for element forces.

4. Change in shape of the propagating velocity wave and its effects on estimation of N_{SYS}

To illustrate the change in shape of the incident velocity wave as it propagates up the height of the building, position of the velocity waves are shown at different instants in Fig. 8, for the case of Structure-II with R of 2 and no viscous damping, subjected to short duration pulse. The figures show the GM velocity, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ curves, which are the $\Delta u/t_h$ waves obtained by three different methods as explained below. The v_y values of selected stories namely 1, 10, 20, 30, and 40 are also shown in the figures.

The GM velocity curve is the $\Delta u/t_h$ wave obtained without any consideration of the change in its magnitude and shape due to the effects of system damping, story yielding, or discontinuity in story stiffness. Also, because the analysis results up to the time taken by the wave to reach the top of the building is of interest, it considers only the forward moving velocity wave with reversed sign, which is equivalent to the ground velocity motion input to the building. Thus, as it travels to the top of the building, it maintains the shape of the full-sine velocity pulse input at the base of the structure.

The $\Delta u_{GM}/t_h$ curve is obtained similarly to the GM velocity curve, by ignoring the effects of damping, story yielding, or discontinuity in story stiffness; however, it is obtained by using the displacement input motion at the base. Consider the displacement excitation input at the base of the structure traveling towards the top as the incident displacement wave without any change in magnitude or wavelength. The difference in the value of the wave at adjacent stories gives the inter-story drift, denoted by Δu_{GM}. If t_h is the time taken by the wave to travel through that story, then the expression $\Delta u_{GM}/t_h$ gives the slope of the displacement curve at that story, which is a term equivalent to the coordinate of a velocity wave at that story. At any given time during structural response, the line joining the value of $\Delta u_{GM}/t_h$ at all the stories is referred to as $\Delta u_{GM}/t_h$ curve. Since $\Delta u_{GM}/t_h$ maintains its shape as it travels up the building, the shape of the curve is same as that of the velocity pulse input at the base.

The $\Delta u_{Analysis}/t_h$ curve is defined as the inter-story drift of a story obtained from OpenSees analysis results, $\Delta u_{Analysis}$, divided by the time taken by the wave to travel through that story, t_h. At any given time during structural response, the line joining the value of $\Delta u_{Analysis}/t_h$ at all the stories is referred to as $\Delta u_{Analysis}/t_h$ curve. Note that since it is derived from the OpenSees analysis results, it will include the effect of changes in amplitude and wavelength due to the effects of damping, inelastic deformations, and changes in element properties. The shape of the $\Delta u_{Analysis}/t_h$ curve will represent the actual shape of the $\Delta u/t_h$ wave traveling through the building as it undergoes changes in amplitude and shape due to various effects.

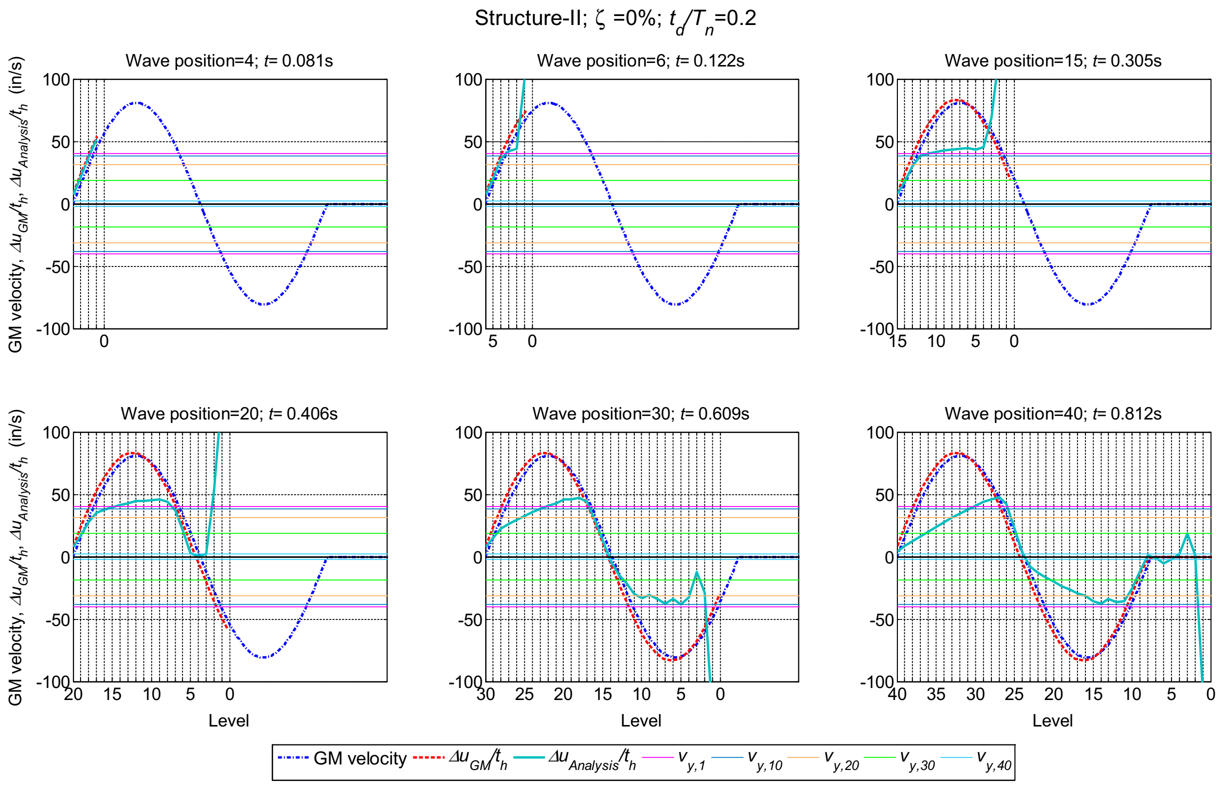

Fig. 8. Position of GM velocity wave, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ at different times for Structure-II with no damping, V_p variation based on code, and R of 2 subjected to short duration base excitation.

Note that $\Delta u/t_h$ is the combination of forward moving velocity wave with negative sign and backward moving velocity wave as expressed in Eq. (1). As explained before, up to the time the wave reaches the top of the building, $\Delta u/t_h$ will only consist of the forward moving velocity wave with reversed sign, which is equivalent to the incident wave (with reversed sign). Thus, in Fig. 8, the GM velocity and $\Delta u_{GM}/t_h$ curves represent the incident velocity wave without change in shape due to the effects of damping, inelasticity, or discontinuity in system property, while $\Delta u_{Analysis}/t_h$ includes all these effects and represents the actual shape of the incident velocity wave propagating up the building height.

At first, when the wave enters the building, the magnitude of the velocity wave is small, the structure is still elastic and the GM velocity, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ curves are aligned. Once the magnitude of the velocity wave exceeds the v_y value of the first story, it causes the story to yield and undergo inelastic deformations. For example, when the wave reaches level 4 at 0.081 s, the magnitude of the incident velocity wave at the first floor exceeds the v_y value at that story; therefore, it yields and undergoes large deformation; this is particularly visible in Fig. 8, as indicated by the large value of $\Delta u_{Analysis}/t_h$ at the first floor when the wave reaches level 6 at 0.122 s. Whereas there is no change in magnitude or wavelength of the GM velocity wave as it does not consider the change in shape due to various effects. Most of the input energy is dissipated at the lower stories through large inelastic deformation. Here large inelastic deformation was found to concentrate at the bottom three stories and hence, they have large $\Delta u_{Analysis}/t_h$ (see Fig. 8 at t = 0.305 s) value. At other stories, the inelastic deformation was small and the $\Delta u_{Analysis}/t_h$ value was closer to their v_y values. As the wave propagates up, inelastic behavior causes attenuation and decreases the magnitude of $\Delta u_{Analysis}/t_h$ close to the v_y value of the story, it has passed as can be seen in Fig. 8 at 0.305 s and onward. Since the v_y values decrease progressively with height in this example (refer to Fig. 7), the wave traveling up still has residual amplitude greater than the v_y values of the stories above it, and it can yield those stories. Hysteretic behavior in these yielded stories will further cause the magnitude of the wave to decrease in those stories. Thus, as the wave progressively moves up, its magnitude continues to progressively decrease.

In case of Structure-IV, that has story stiffness decreasing along the height, it will cause the waves to get compressed as they move up. Thus, the magnitude of the $\Delta u_{Analysis}/t_h$ curve was found to increase as it moves up the building. In general, for Structure-IV with viscous damping, there are two phenomena affecting the magnitude (and hence the shape) of the traveling wave: first, a decrease in the magnitude of the wave occurs as the wave moves up, due to damping and attenuation caused by the inelastic behavior, and; second, an increase in the magnitude of the wave occurs due to the decrease in story stiffness as the wave travels up.

It was observed that as the velocity waves propagates forward, the part of the $\Delta u_{Analysis}/t_h$ wave, above the v_y value of the stories it has passed changes but the part below it does not change. For example, when the wave reaches 20th story at 0.406 s, as shown in Fig. 8 (a close-up view of this figure is shown in Fig. 9), the part of the wave approximately above the v_y values of this story has changed; but the part of the wave below this has not changed and maintains its original form. It is this part of the wave that determines the time at which the yielding of this story starts and ends. Thus, for the shear-type structures with v_y values decreasing with height considered for the study, the change in shape of the wave due to the inelastic behavior does not significantly affect the duration of story yielding (i.e., start and end times of story yielding), and hence it will also not affect the prediction of the number of simultaneously yielding stories. Therefore, the

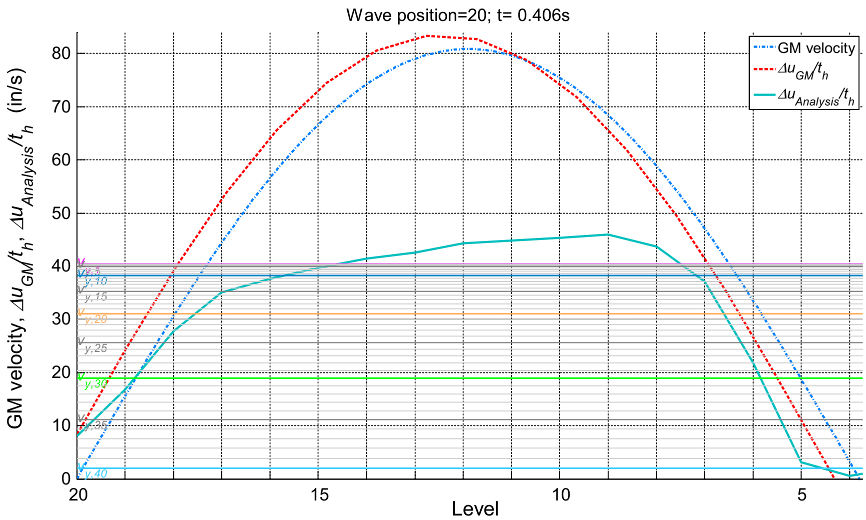

Fig. 9. Closer view of the velocity waves at the instant when the wave reaches the 20th level at 0.406 s for Structure-II with no viscous damping, variation of V_p values based on code, and R of 2 subjected to short duration (t_d/T_n = 0.2) pulse base excitation.

velocity base excitation can be used to predict the number of simultaneously yielding stories for the structures considered here.

5. Predicting number of simultaneously yielding stories using GM velocity

As mentioned before, the proposed procedure for predicting the number of simultaneous yielding stories due to the incident wave considers the velocity wave input at the base, termed as GM velocity, propagating along the building height in the form of an incident velocity wave without any change to its amplitude and wavelength. To verify the estimated values, the results obtained are compared with those obtained using the $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ curves and then checked against the story yield time history obtained from the OpenSees analysis results that shows the stories that actually yield at a given time. Comparison of yielding predicted using the GM velocity curve to the actual yielding obtained using the $\Delta u_{Analysis}/t_h$ curve and from the story yield history will provide a measure of the accuracy of the proposed procedure.

To illustrate the procedure for calculating the number of stories yielding simultaneously, consider (for example) Structure-II with no viscous damping and a V_p distribution based on the code prescribed lateral force distribution, subjected to a full-sine velocity pulse with t_d/T_n equal to 0.2 and R of 2. Consider an instant when the $\Delta u/t_h$ wave has reached the 20th floor of the building at 0.406 s, as shown in Fig. 8. A closer view of the GM velocity, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ curves at that instant are shown in Fig. 9 focusing on the first part of the velocity wave. Fig. 10 shows the story yield time history for the structure being considered. The red and blue dots indicate story yielding. Since the time interval is small, the dots overlap and resemble horizontal bars. As the wave propagates up the building, story yielding can be seen to progress along. The first inclined red pattern is due to the first part of the wave and the blue pattern is due to the second part of the wave. Time history of the total number of simultaneously yielding stories, as the wave propagates up the building, is also shown in the figure. Figs. 9 and 10 will be used to obtain the number of story yielding simultaneously based on the four methods mentioned before; Fig. 9 will be used to obtain N_{SYS} values based on GM velocity, $\Delta u_{GM}/t_h$ curve, and $\Delta u_{Analysis}/t_h$ curve, and Fig. 10 will be used to obtain N_{SYS} value based on story yield history obtained from OpenSees analysis as follows:

- *GM velocity:* Results show that stories 7 through 17 have v_y values smaller than the value of the GM velocity curve at those corresponding levels, and therefore will undergo yielding simultaneously (as shown in Fig. 9).
- *$\Delta u_{GM}/t_h$ curve:* Comparing the v_y value of a story and the value of the $\Delta u_{GM}/t_h$ curve at that corresponding story, stories 7 to 18 are predicted to yield (as shown in Fig. 9).
- $\Delta u_{Analysis}/t_h$ curve: Comparing the v_y value of the stories and the magnitude of the $\Delta u_{Analysis}/t_h$ curve, stories 8 to 17 are observed to actually undergo yielding (as shown in Fig. 9).
- *Story yield history:* Based on the story shear force obtained from the actual time history analysis of the structure performed in OpenSees, yielding is observed in stories 9 through 17 when the wave reaches the 20th story at 0.406 s (as shown in Fig. 10), for a total of 9 simultaneously yielding stories (note that, in those results, yielding is defined as when the shear force in a particular story is greater than its yield capacity).

The number of simultaneously yielding stories predicted using GM velocity and $\Delta u_{GM}/t_h$ are 11 and 12, respectively, while the actual stories yielding simultaneously obtained using $\Delta u_{Analysis}/t_h$ and story yield history, for verification purpose, are 10 and 9, respectively. Thus, estimation of the number of simultaneous yielding stories using the proposed procedure for a pulse of ground velocity input is found to give a good approximation of the actual number of story yielding simultaneously.

The results for number of simultaneously yielding stories obtained using the four methods mentioned are presented in Tables 2 and 3 for V_p variation based on code (note that here results for R of 4 and 6 are only shown). The results show that for this variation of V_p values, except for the case of Structure-IV subjected to long duration pulse, the number of story yielding predicted using the GM velocity and $\Delta u_{GM}/t_h$ curve are close to the actual yielding verified from the $\Delta u_{Analysis}/t_h$ curve and story yield history (obtained from OpenSees analysis results). Not only do they closely match the actual total number of stories yielding simultaneously, but are in agreement as to the location of yielding over the building height. The results differ by only one or two stories in few cases. For Structure-IV subjected to long duration pulse, the number of simultaneously yielding stories obtained from GM velocity, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ agree as to the total number and location of yielded stories, but differ from the actual number of story yielding obtained from OpenSees analysis which shows a break in the

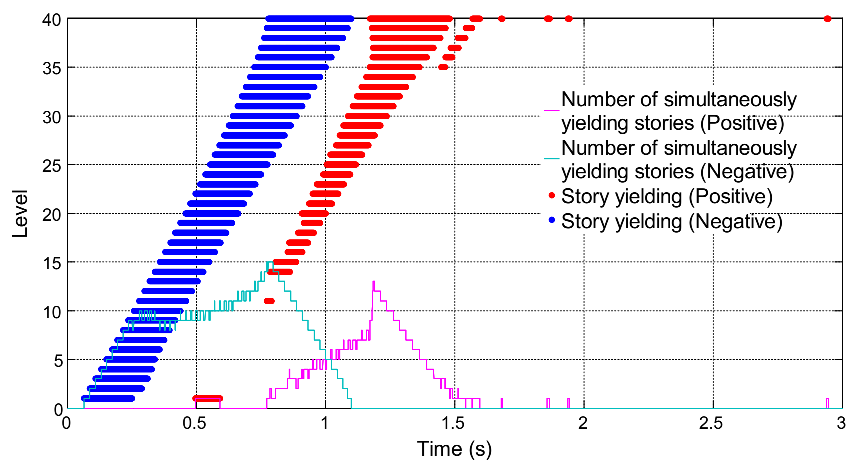

Fig. 10. Story yield time history for Structure-II with no viscous damping and variation of V_p values based on code with R of 2, subjected to short duration pulse base excitation ($t_d/T_n = 0.2$).

Table 2
Simultaneously yielding stories for Structure-II with 2% damping and V_p based on code specified lateral force distribution subjected to short and long duration pulses.

Structure type	ζ	t_d/T_n	R	Wave position	Time (s)	Number of simultaneously yielding stories			
						Based on GM velocity	Based on $\Delta u_{GM}/t_h$ curve	Based on $\Delta u_{Analysis}/t_h$ curve	Based on story yielding history
Structure-II	2%	0.2	4	5	0.102	1–3 (3)	1–4 (4)	1–3 (3)	1–3 (3)
				10	0.203	1–8 (8)	1–9 (9)	1–7 (7)	1–8 (8)
				15	0.305	1–13 (13)	1–14 (14)	1–12 (12)	3–12 (10)
				20	0.406	6–19 (14)	6–19 (14)	7–17 (11)	8–17 (10)
				25	0.508	11–24 (14)	11–24 (14)	12–23 (12)	13–23 (11)
				30	0.609	15–29 (15)	16–29 (14)	17–28 (12)	18–28 (11)
				35	0.711	20–34 (15)	21–35 (15)	21–35 (15)	23–35 (13)
				40	0.812	25–39 (15)	26–40 (15)	25–40 (16)	27–40 (14)
			6	5	0.102	1–4 (4)	1–4 (4)	1–3 (3)	1–3 (3)
				10	0.203	1–9 (9)	1–9 (9)	1–8 (8)	1–8 (8)
				15	0.305	1–14 (14)	1–14 (14)	1–13 (13)	2–13 (12)
				20	0.406	5–19 (15)	6–19 (14)	4–18 (15)	7–18 (12)
				25	0.508	10–24 (15)	11–24 (14)	11–23 (13)	12–23 (12)
				30	0.609	15–29 (15)	15–30 (16)	16–29 (14)	17–29 (13)
				35	0.711	20–34 (15)	20–35 (16)	20–35 (16)	22–35 (14)
				40	0.812	25–39 (15)	25–40 (16)	25–40 (16)	26–40 (15)
Structure-II	2%	0.8	4	5	0.102	–	–	–	–
				10	0.203	1–4 (4)	1–5 (5)	1–4 (4)	1–4 (4)
				15	0.305	1–10 (10)	1–10 (10)	1–9 (9)	1–9 (9)
				20	0.406	1–15 (15)	1–16 (16)	1–16 (16)	1–15 (15)
				25	0.508	1–21 (21)	1–22 (22)	1–21 (21)	1–21 (21)
				30	0.609	1–27 (27)	1–27 (27)	1–27 (27)	1–27 (27)
				35	0.711	1–33 (33)	1–33 (33)	1–33 (33)	1–32 (32)
				40	0.812	1–39 (39)	1–40 (40)	1–40 (40)	1–40 (40)
			6	5	0.102	1 (1)	1–2 (2)	1 (1)	1 (1)
				10	0.203	1–6 (6)	1–7 (7)	1–6 (6)	1–6 (6)
				15	0.305	1–11 (11)	1–12 (12)	1–11 (11)	1–11 (11)
				20	0.406	1–17 (17)	1–17 (17)	1–16 (16)	1–16 (16)
				25	0.508	1–22 (22)	1–23 (23)	1–22 (22)	1–22 (22)
				30	0.609	1–28 (28)	1–28 (28)	1–27 (27)	1–27 (27)
				35	0.711	1–33 (33)	1–34 (34)	1–34 (34)	1–34 (34)
				40	0.812	1–39 (39)	1–40 (40)	1–40 (40)	1–40 (40)

sequence of simultaneously yielding stories. For example, for R of 4, when the wave reaches the 35th story at 0.772 s (which corresponds to the 23rd row with bold text in Table 3), the GM velocity, $\Delta u_{GM}/t_h$, and $\Delta u_{Analysis}/t_h$ results predict that yielding will occur over stories 1 to 33. However, OpenSees analysis results reveal yielding occurs in stories 1 to 9 and from 19 to 33; there was no simultaneous yielding observed in the 10–18th stories. This discontinuity in the OpenSees results was partly due to some fluctuations in the shear force history, producing slight unloading prior to reloading observed over the 10-18th stories, with the absence of yielding being simply due to the temporary dip of the story force below the yield strength of the story. When the strain hardening ratio was increased, or when a tolerance value for the definition of shear yield capacity was considered in the definition of story yield, results gave a continuous yield pattern and the number of stories yielding and their location obtained from the four methods were in agreement.

6. Mathematical formulation for predicting simultaneously yielding stories

With the information on the geometry of the velocity base excitation and the v_y values of the stories, mathematical expression for

Table 3
Simultaneously yielding stories for Structure-IV with 2% damping and V_p based on code specified lateral force distribution subjected to short and long duration pulses.

Structure type	ζ	t_d/T_n	R	Wave position	Time (s)	Number of simultaneously yielding stories: Based on GM velocity	Based on $\Delta u_{GM}/t_h$ curve	Based on $\Delta u_{Analysis}/t_h$ curve	Based on story yielding history
Structure-IV	2%	0.2	4	5	0.092	1–3 (3)	1–4 (4)	1–3 (3)	1–3 (3)
				10	0.185	1–8 (8)	1–9 (9)	1–7 (7)	1–7 (7)
				15	0.281	1–13 (13)	1–14 (14)	1–12 (12)	2–12 (11)
				20	0.382	5–18 (14)	5–19 (15)	6–17 (12)	9–19 (11)
				25	0.493	11–24 (14)	11–24 (14)	12–23 (12)	14–23 (10)
				30	0.618	17–29 (13)	18–29 (12)	18–28 (11)	20–28 (9)
				35	0.772	24–34 (11)	25–34 (10)	25–35 (11)	26–34 (9)
				40	1.043	34–39 (6)	35–40 (6)	34–40 (7)	36–40 (5)
			6	5	0.092	1–4 (4)	1–4 (4)	1–3 (3)	1–3 (3)
				10	0.185	1–9 (9)	1–9 (9)	1–8 (8)	1–8 (8)
				15	0.281	1–14 (14)	1–14 (14)	1–13 (13)	2–13 (12)
				20	0.382	4–19 (16)	5–19 (15)	4–18 (15)	6–18 (13)
				25	0.493	11–24 (14)	11–24 (14)	12–23 (12)	13–23 (11)
				30	0.618	17–29 (13)	17–30 (14)	18–28 (11)	19–28 (10)
				35	0.772	24–34 (11)	24–35 (12)	24–34 (11)	26–34 (9)
				40	1.043	34–39 (6)	35–40 (6)	34–40 (7)	35–40 (6)
Structure-IV	2%	0.8	4	5	0.092	–	–	–	–
				10	0.185	1–4 (4)	1–5 (5)	1–4 (4)	1–4 (4)
				15	0.281	1–10 (10)	1–10 (10)	1–9 (9)	1–9 (9)
				20	0.382	1–15 (15)	1–16 (16)	1–15 (15)	1–15 (15)
				25	0.493	1–21 (21)	1–21 (21)	1–21 (21)	1–7,9–21 (20)
				30	0.618	1–27 (27)	1–27 (27)	1–27 (27)	1–8,14–27 (22)
				35	**0.772**	**1–33 (33)**	**1–33 (33)**	**1–33 (33)**	**1–9,19–33 (24)**
				40	1.043	1–39 (39)	1–40 (40)	1–40 (40)	6–12,24–40 (24)
			6	5	0.092	1 (1)	1–2 (2)	1 (1)	1 (1)
				10	0.185	1–6 (6)	1–7 (7)	1–6 (6)	1–6 (6)
				15	0.281	1–11 (1)	1–12 (12)	1–11 (11)	1–11 (11)
				20	0.382	1–16 (16)	1–17 (17)	1–16 (16)	1–17 (17)
				25	0.493	1–22 (22)	1–23 (23)	1–22 (22)	1–8,10–22 (21)
				30	0.618	1–28 (28)	1–28 (28)	1–28 (28)	1–9,13–27 (24)
				35	0.772	1–33 (33)	1–34 (34)	1–34 (34)	1–10,18–34 (27)
				40	1.043	1–39 (39)	1–40 (40)	1–40 (40)	4–12,24–40 (25)

finding the time at which yielding at a particular story begins and ends can be obtained. With this, equations of the two lines, named t_C and t_D, that envelop the story yielding at different stories, caused by the incident wave traveling up the building can be derived. Using these two lines, the number of stories yielding simultaneously at any given time before the incident wave reaches to the top of the building (i.e. simultaneous yielding due to an incident wave) can be found. These mathematical expressions that can be used to predict the number of simultaneously yielding stories in shear-type building model considered, have been derived by two methods: first, equations for lines t_C and t_D are derived based on the full-sine velocity wave used as base excitation; next, the equations are derived assuming that the base excitation is rectangular in shape, in an effort to obtain a closed form solution that directly gives a total number of simultaneously yielding stories due to the incident wave.

6.1. Considering full-sine pulse velocity base excitation

Consider a building subjected to a full-sine velocity base excitation, as shown in Fig. 11a. For time t less than the pulse duration t_d, the equation of a full-sine velocity base excitation, as shown in Fig. 11a, can be written as:

$$v_g(t) = v_{g0} \sin \omega_d t \tag{30}$$

where v_{g0} is the amplitude and w_d is the radial frequency of the full-sine velocity wave with time period t_d. Alternately, Eq. (30) can also be written as:

$$t = \frac{1}{\omega_d} \sin^{-1}\left(\frac{v_g(t)}{v_{g0}}\right) \tag{31}$$

Horizontal lines representing the minimum magnitude of velocity wave required to yield different stories in the building (the v_y values) have also been shown in Fig. 11a. Let v_{yj} denote the minimum magnitude of the velocity base excitation required to yield the j^{th} story. When the value of the incident wave at the j^{th} story is greater than the v_{yj} value, it will yield that story. Thus, in the example shown, if the part of the incident velocity wave between time duration t_{Aj} and t_{Bj} falls within the j^{th} story, there will be yielding in that story. The points along the velocity curve, corresponding to t_{Aj} and t_{Bj} have been marked as A and B. The value of t_{Aj} can be obtained by substituting v_{yj} in place of $v_g(t)$ in Eq. (31). Thus,

$$t_{Aj} = \frac{1}{\omega_d} \sin^{-1}\left(\frac{v_{yj}}{v_{g0}}\right) \tag{32}$$

The value of t_{Bj} can then be obtained as:

$$t_{Bj} = \frac{t_d}{2} - t_{Aj} = \frac{t_d}{2} - \frac{1}{\omega_d} \sin^{-1}\left(\frac{v_{yj}}{v_{g0}}\right) \tag{33}$$

The values of t_{Aj} and t_{Bj} represent the time at which the two points A and B respectively reach the base of the building. As the incident wave moves up, when point A reaches the j^{th} story, the yielding in that story will begin. The time taken by point A to reach the j^{th} story is denoted as t_{Cj} and will be equal to t_{Aj} plus the time the wave takes to travel from the base to that floor, as shown in Eq. (34).

$$t_{Cj} = t_{Aj} + \sum_{i=1}^{j} t_{hi} = \frac{1}{\omega_d} \sin^{-1}\left(\frac{v_{yj}}{v_{g0}}\right) + \sum_{i=1}^{j} t_{hi} \tag{34}$$

In Eqs. (34), t_{hi} represents the time taken by the wave to travel through the i^{th} story. If c_i denotes the velocity of the traveling wave

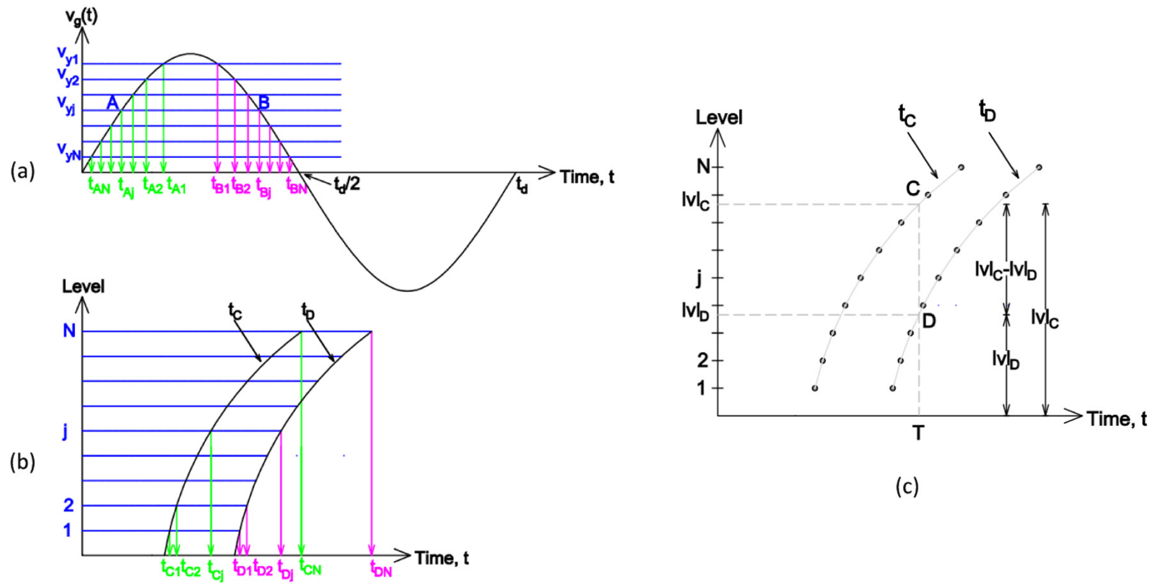

Fig. 11. (a) Point A and B along full-sine velocity base excitation corresponding to beginning and end of story yielding; (b) the t_C and t_D curves indicating the beginning and end of story yielding, (c) estimating number of simultaneously yielding stories at time T due to full-sine velocity pulse base excitation.

at the i^{th} story with height h_i and story stiffness k_i, the time taken by the wave to travel through that story can be calculated as:-

$$t_{hi} = \sqrt{\frac{h_i\,\bar{m}_i}{k_i}} \tag{35}$$

Yielding at the j^{th} story will stop after point B reaches that story. The time taken by point B to reach the j^{th} story is denoted by t_{Dj} and can be obtained by taking the summation of t_{Bj} and the time taken by the wave to reach the j^{th} story. Thus,

$$t_{Dj} = t_{Bj} + \sum_{i=1}^{j} t_{hi} = \frac{t_d}{2} - \frac{1}{\omega_d}\sin^{-1}\left(\frac{v_{yj}}{v_{g0}}\right) + \sum_{i=1}^{j} t_{hi} \tag{36}$$

Note that as mentioned before, point B is an approximate location along the velocity curve that corresponds to the end of story yielding caused by the incident wave. Thus, t_{Dj} indicates the approximate time at which the story yielding due to the incident wave ends. Furthermore, since v_y values decreases with height for the structure considered, when point B passes through the stories below the j^{th} stories, those stories would have stopped yielding and would probably be in the unloading path of the hysteretic curve. Thus, t_{hi} values in Eq. (36) can be obtained using Eq. (35).

Note that t_{Aj}, t_{Bj}, t_{Cj}, and t_{Dj} exists only if $v_{yj} \leqslant v_{g0}$. Thus, the t_{Cj} and t_{Dj} values can be obtained for all the stories that have v_{yj} values less than or equal to the magnitude of the velocity wave v_{g0}. With these values, the t_C and t_D curves that mark the beginning and end of story yielding at different stories can be plotted as shown in Fig. 11b. Once t_C and t_D are plotted or the t_{Cj} and t_{Dj} values at all levels are known, the data can be used to find number of stories yielding at any time. For example, in Fig. 11c, using the already available information on t_{Cj} and t_{Dj} at each level, the data can be interpolated to find the points $C(T, lvl_C)$ and $D(T, lvl_D)$ along the t_C and t_D curve corresponding to a given time 'T'. Points D and C correspond to the two locations along the building between which the simultaneous story yielding occurs at time 'T'. The story levels corresponding to points C and D are marked lvl_C and lvl_D. Once lvl_C and lvl_D are known the difference between the two will give the number of stories yielding at time T as show in Eq. (37).

$$N_{SYS} = lvl_C - lvl_D \tag{37}$$

6.2. Considering rectangular pulse velocity base excitation

Here, by assuming that a velocity base excitation is rectangular in shape as shown in Fig. 12a, a closed form solution for estimating the number of simultaneously yielding stories is obtained. To simplify the expressions, the structure is also assumed to have: (i) constant story weight and height at all the stories in the building, and; (ii) variation of story stiffness over the building height based on the lateral force distribution as prescribed by code procedures. As a result, distribution of the shear forces and the story stiffness is parabolic over the height of the building. Also for the shear type building considered here, the inter-story drift at all the stories of the building are equal.

The continuous formulation for t_{Cj} and t_{Dj}, denoted as $t_C(x)$ and $t_D(x)$ respectively (refer to Fig. 12b), can be expressed as (refer to Shrestha and Bruneau [17] for derivation):

$$t_C(x) = 0 + \sqrt{\frac{\bar{m}H^2}{hk_b}}\sin^{-1}\frac{x}{H} \tag{38}$$

$$t_D(x) = \frac{t_d}{2} + \sqrt{\frac{\bar{m}H^2}{hk_b}}\sin^{-1}\frac{x}{H} \tag{39}$$

Let the point along t_C curve at a particular time T be denoted by $C(T,x_C)$ and that along t_D curve be $D(T,x_D)$, as shown in Fig. 12c. Then, using Eqs. (38) and (39), x_C and x_D can be obtained as:

$$x_C = \begin{cases} H\sin\left(\sqrt{\frac{hk_b}{\bar{m}H^2}}T\right) & t_{C,\min} \leqslant T \leqslant t_{C,\max} \\ x_{y,\max} & t_{C,\max} \leqslant T \leqslant t_{D,\max} \end{cases} \tag{40}$$

and,

$$x_D = \begin{cases} x_{y,\min} & t_{C,\min} \leqslant T \leqslant t_{D,\min} \\ H\sin\left\{\sqrt{\frac{hk_b}{\bar{m}H^2}}\left(T - \frac{t_d}{2}\right)\right\} & t_{D,\min} \leqslant T \leqslant t_{D,\max} \end{cases} \tag{41}$$

where $x_{y,\min}$ denotes the distance from the base of the building to the lowermost story with v_y value less than the amplitude of the incident velocity wave; while, $t_{C,\min}$ and $t_{D,\min}$ respectively denote the t_{Cj} and t_{Dj} values for that story. Similarly, $x_{y,\max}$ denotes the

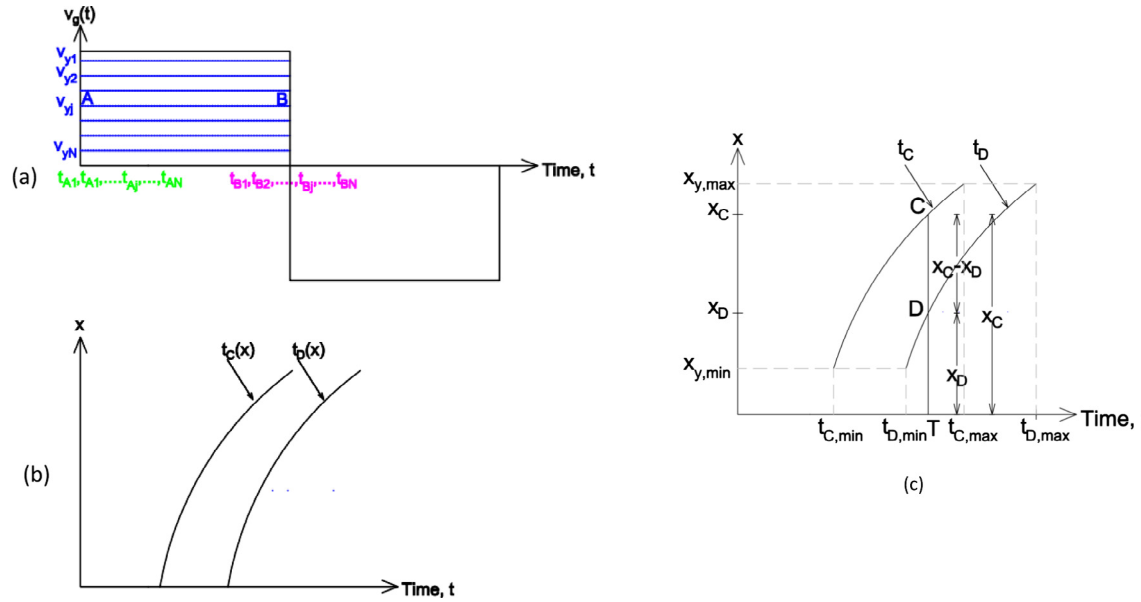

Fig. 12. (a) Point A and B along rectangular velocity pulse corresponding to the beginning and end of story yielding; (b) the t_C and t_D curves indicating the beginning and end of story yielding and (c) estimating number of simultaneously yielding stories at time *T*.

distance from the base of the building to the uppermost story with v_y value less than the amplitude of the incident velocity wave; while, $t_{C,\max}$ and $t_{D,\max}$ respectively denote the t_{Cj} and t_{Dj} values for that story. Using Eqs. (40) and (41), x_C and x_D value can be found for anytime *T* and the estimated number of simultaneously yielding stories N_{SYS} can be obtained using Eq. (42).

$$N_{SYS}(T) = \frac{x_C - x_D}{h} \tag{42}$$

Since some assumptions, mentioned above, were made on distribution of story stiffness, height, and weight to simplify the derivation process, application of this procedure in other distribution of these parameters may introduced some discrepancy in the estimated N_{SYS} values.

6.3. Application of mathematical formulation to estimate the N_{SYS} values

Application of the mathematical formulation for predicting the N_{SYS} value for the structure with V_p variation based on code considered showed that the equations for the t_C and t_D curves considering the full-sine velocity pulse give good match with the beginning and end of the story yielding obtained from the OpenSees analysis results for both Structures-II and -IV, subjected to short duration pulse (Figs. 13 and 14 respectively). Consequently, the total number of simultaneously yielding stories estimated was also close to the actual values. Note that for the structure with $R = 1$, which is defined such that the shear yield capacity V_p at the base is equal to the maximum elastic demand in that story, the shear yield

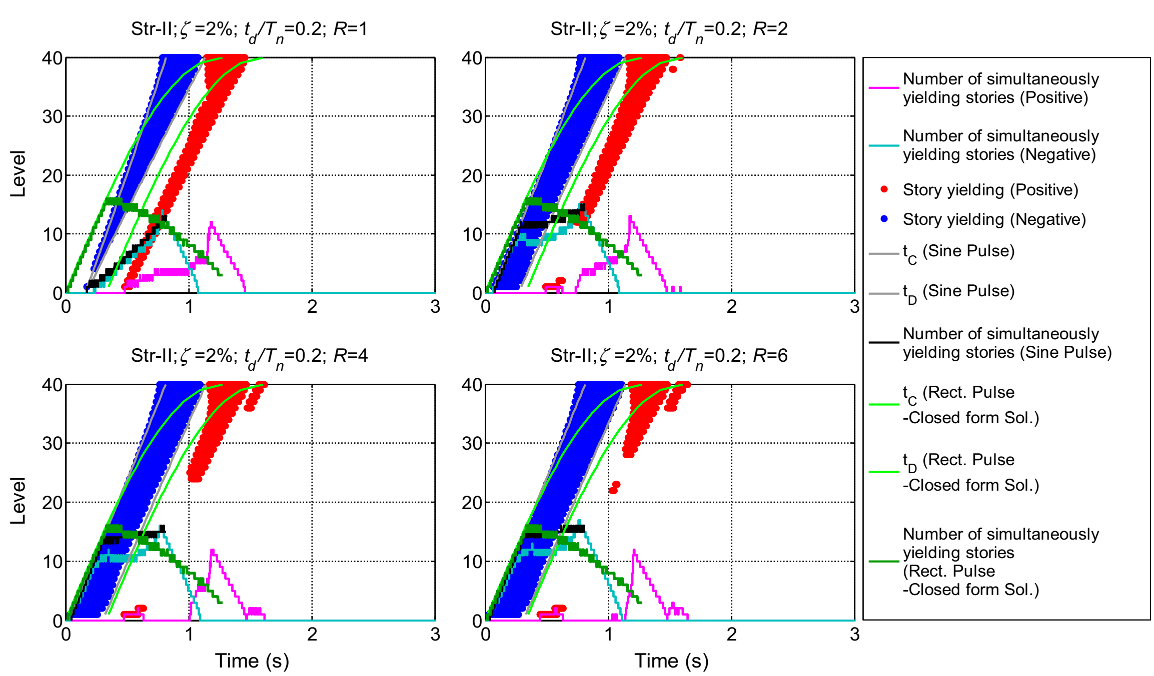

Fig. 13. Estimation of number of simultaneously yielding stories for Structure-II with *R* of 1, 2, 4, and 6 subjected to short duration ($t_d/T_n = 0.2$) full-sine velocity pulse base excitation.

capacity at the other stories is calculated based on a parabolic distribution of V_p values over the building height (much like what would result from a building code approach). Hence, in that case, $R = 1$ at the base story, but at the other stories it may be different as demands along the height may not perfectly match this assumed strength distribution, and yielding does occur at the stories that have an "effective" R greater than one.

In case of long duration pulse, the t_C curve matched well with the beginning of yielding at each story observed from OpenSees analysis result, but there was a gap between the t_D curve and the end of story yielding observed from OpenSees results. This was due to the assumption made for the location of the point along the velocity curve where story yielding ends. In Structure-II with long duration pulse (refer to Fig. 15), some of the stories have shorter yield duration as compared to other stories in the building. This is due to the fluctuation in story force observed in the OpenSees analysis results. The shortening of the story yield duration in the upper stories is considered to be due to the overlapping of the

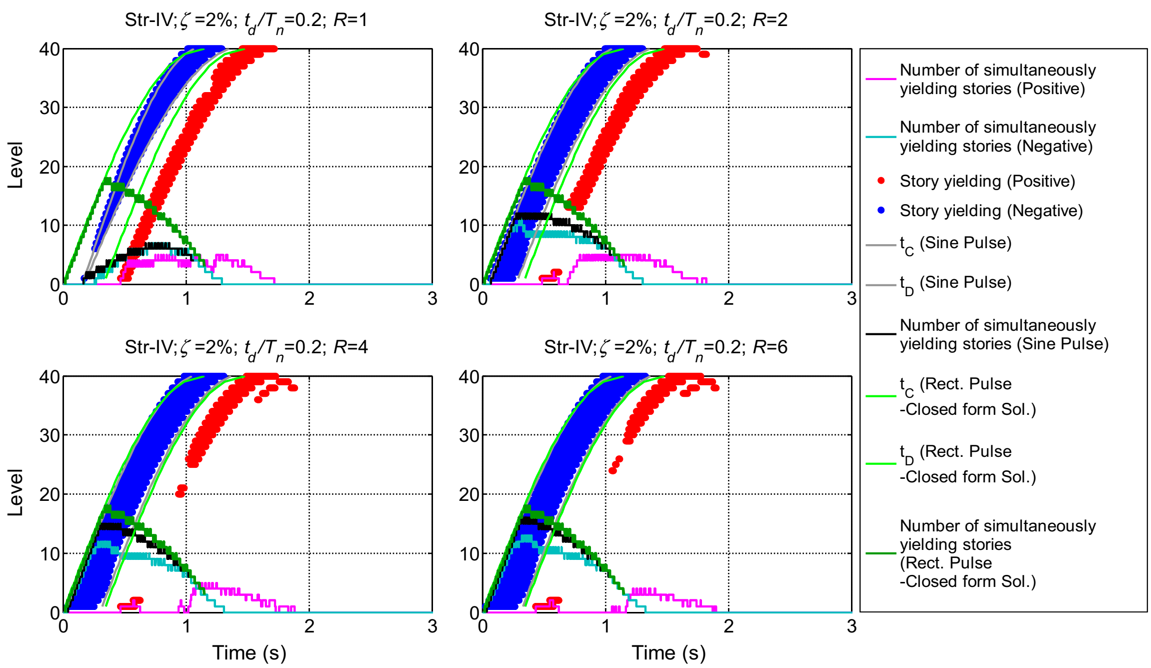

Fig. 14. Estimation of number of simultaneously yielding stories for Structure-IV with R of 1, 2, 4, and 6 subjected to short duration ($t_d/T_n = 0.2$) full-sine velocity pulse base excitation.

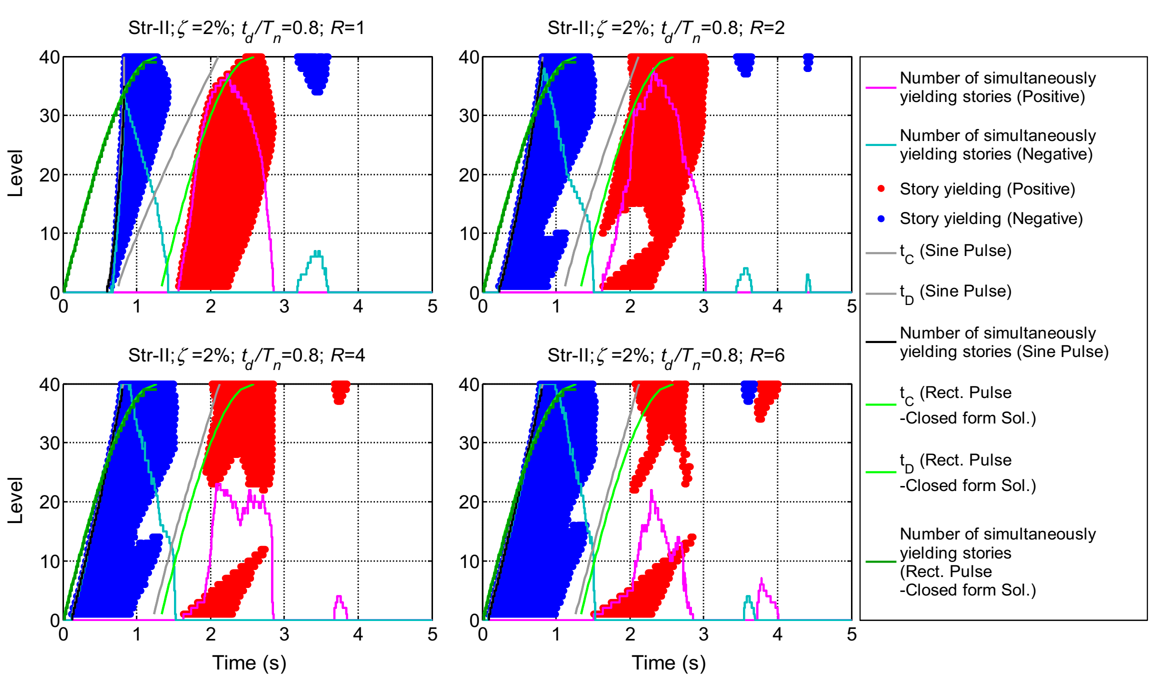

Fig. 15. Estimation of number of simultaneously yielding stories for Structure-II with R of 1, 2, 4, and 6 subjected to long duration ($t_d/T_n = 0.8$) full-sine velocity pulse base excitation.

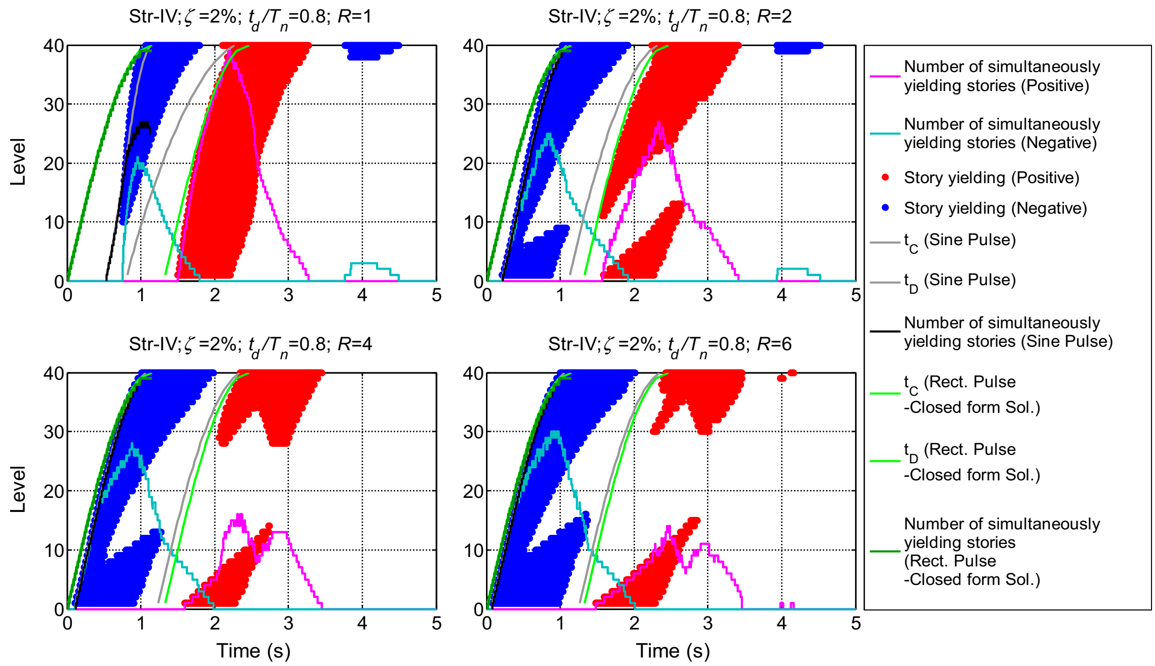

Fig. 16. Estimation of number of simultaneously yielding stories for Structure-IV with R of 1, 2, 4, and 6 subjected to long duration ($t_d/T_n = 0.8$) full-sine velocity pulse base excitation.

velocity waves that takes place over a large span. In Structure-IV with long duration pulse (refer to Fig. 16), there is also a distinct shortening of the yield duration and non-uniformity in the yield pattern. An additional factor contributing to the observation in this case is the refraction and reflection of the velocity wave due to the discontinuity of story stiffness at the floor. These factors create additional discrepancies between the estimated and the actual number of simultaneously yielding stories. However, these discrepancies were found to significantly attenuate when hysteretic behavior with strain-hardening is taken into account, instead of an elastic-ideal-plastic one, even when some tolerance in the yield values was considered (Shrestha and Bruneau [17]). Estimated N_{SYS} values were close to the actual values for Structure-II, with three instances where the estimated value was less than the actual values with a discrepancy of 1. For Structure-IV the estimation procedure provided conservative results especially in the case of long duration pulse.

As expected, the total number of stories yielding simultaneously estimated using the method for predicting the N_{SYS} values that considered a rectangular velocity base excitation pulse instead of a full-sine velocity pulse was found to slightly overestimate the actual number, but the resulting conservatism is considered to be within an acceptable range.

7. Conclusion

Concepts of wave propagation in elastic medium were used in an attempt to find a systematic procedure for estimating the number of simultaneously yielding stories, which is in itself information needed to eventually estimate the axial force demand in the columns of seismic force resisting systems. Towards that goal, the initial step of finding the number of simultaneously yielding stories in a shear building subjected to full-sine velocity base excitation has been presented here. Story yielding was found to be related to the velocity waves (resulting from base excitation) propagating along the building height.

The v_y values for the stories of a building, that depend on the system properties (shear yield capacity, story stiffness, mass, and height) was found to be the parameter by which it is possible to determine if a velocity wave input can cause yielding in the stories of a shear-type building. Story yielding due to a base excitation would occur if the resultant magnitude of the forward moving velocity wave with reverse sign and a backward moving velocity wave exceeded or equaled the v_y values of the story. For the shear building model considered for the study, as the incident velocity wave propagates up the building, the part of the wave above the v_y values of the stories it has passed was found to change, but the part below the v_y value did not change. Hence, for the structures considered here with v_y values decreasing with height, estimating the number of simultaneously yielding stories due to incident wave using the velocity base excitation without considering the change in shape and size provided estimation results that matched well with the actual results obtained from the OpenSees analysis. Therefore, assuming that the velocity wave did not change in its shape or magnitude as it propagates was found to be adequate for predicting the number of simultaneous yielding stories for the structures considered here (i.e., the number of simultaneously yielding stories, hence, could be adequately estimated without considering a wave propagation formulation in inelastic medium).

Mathematical formulations for finding the N_{SYS} values that considered the sinusoidal shape of the input motion for prediction of story yielding was found to work well for structures subjected to short duration pulse. For long duration pulse, there was small discrepancy between the predicted end of story yielding and that observed from OpenSees analysis, due to the assumption made in defining the end of story yielding. Due to this discrepancy the estimation N_{SYS} values were slightly conservative compared to the actual values. The prediction method that assumes rectangular shape was found to provide conservative results within an acceptable range.

8. Future research

As a continuation of this study, the next steps would be to extend the estimation procedure developed here (for a shear-type building subjected to full-sine velocity pulse base excitation) to the case when such buildings are subjected to actual earthquakes, by representing the pulses in their velocity record by equivalent full-sine velocity pulse. Additionally, formulation of a procedure for calculating the axial force demands by incorporating the N_{SYS} values obtained from the procedures developed previously should be sought. Thereafter, future research can investigate how the study can further be extended to include flexural behavior that exists in tall structures and address issues related to more complicated structures; this would include application of the estimation procedure on lateral-forces resisting frames such as EBF and BRBF.

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